5/H-28 (v) (Syllabus–2015)

2 0 1 7

(October)

STATISTICS

(Honours)

(Mathematical Methods and Distribution Theory)

[STH–51 (TH)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* State and prove Weddle's rule of numerical integration. 6

(b) Describe Newton-Raphson method of solving numerical equation. 6

2. (a) Describe Jacobian of transformation. For two-dimensional continuous random variables X and Y having joint probability density function

$$f(x, y) = \begin{cases} 4xye^{-(x^2+y^2)} & ; \quad x \geq 0, y \geq 0 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Find the p.d.f of $U = \sqrt{X^2 + Y^2}$. 6

(b) Define beta and gamma integrals. Prove that

$$\beta(l, m) = \frac{\Gamma l \, \Gamma m}{\Gamma(l+m)}$$ 6

UNIT—II

3. (a) Define linear dependence and independence of vectors with examples of each. 5

(b) Define rank of a matrix. If A and B are two n-rowed square matrices, then show that Rank $(AB) \geq$ Rank (A) + Rank $(B) - n$. 6

4. (a) Define eigenvalues and eigenvectors. Show that the eigenvalues of A and A^t (where A^t is the transpose of matrix A) are the same. 5

(b) State and prove Cayley-Hamilton theorem. 6

UNIT—III

5. (a) Define marginal and conditional distribution functions. Write the properties of joint distribution function. 6

(b) The joint p.d.f. of two random variables X and Y is given by

$$f(x, y) = \frac{9(1+x+y)}{2(1+x)^4(1+y)^4}; \quad \begin{matrix} 0 \leq x < \infty \\ 0 \leq y < \infty \end{matrix}$$

Find the marginal distribution of X. 5

6. (a) Define conditional expectation and conditional variance of random variables. For the two-dimensional random variables X and Y, show that 5

$$V(X) = E\,[V(X|Y)] + V\,[E(Y|X)]$$

(b) Define the following and also write their properties : 6

(i) Moment generating function

(ii) Cumulant generating function

(iii) Characteristic function

UNIT—IV

7. (a) Define negative binomial distribution. Obtain moment generating function of negative binomial distribution and hence compute its mean and variance. 6

(b) If X and Y are independent gamma variates with parameters μ and ν respectively, show that the variables

$$U = X + Y, \; Z = \frac{X}{X+Y}$$

are independent and U is a $\gamma(\mu + \nu)$ variate and Z is a $\beta(\mu, \nu)$ variate. 5

8. *(a)* Explain in brief how you will utilize hypergeometric distribution to estimate the number of fishes from a pond. 4

(b) Define log-normal distribution and hence obtain its mean and variance. Write the importance of log-normal distribution. 7

UNIT—V

9. *(a)* What do you mean by sampling distribution? Based on a random sample of size n from a normal population having mean μ and variance σ^2, find the sampling distribution of sample mean. 5

(b) Define Chi-square variate and derive its distribution. 6

(Continued)

10. *(a)* Define Student's t statistic and derive its probability density function. 5

(b) Define F-statistic and mention its properties. Establish the relationship between F and χ^2 distributions. 6

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